

Nonlinear Temperature-Dependent Transfer Characteristics of Silicon Photonic Microring Resonators

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Abstract: We investigate the nonlinear power transfer characteristics of a silicon photonic microring resonator. We show that the microring device behaves as a power limiter if the roll-off tail of its resonance is steeper than a simple Lorentzian lineshape.

OCIS codes: (230.4110) Modulators; (230.4555) Coupled Resonators; (200.4650) Optical Interconnects.

1. Introduction

Silicon microring resonators [1] are among the promising options for realizing on-chip and chip-to-chip optical interconnects. They have been demonstrated to operate as modulators [2,3], demultiplexers [3], as well as spatial switches [4], in a limited area footprint. These attributes have made microring resonators very attractive to system designers [5]. Fabrication processes for high- Q ring resonators have matured in the recent years, and thermal stabilization of the rings, required to compensate temperature variations, has also substantially progressed [1,6,7].

Due to the strong light-confinement nature of the microring resonators, the optical power density inside the ring can rapidly build up. Since the exchange of energy with the surrounding through heat conduction is not instantaneous, the high power density leads to two-photon absorption and other nonlinear absorption processes, which raise the ring temperature. This temperature change affects the medium refractive index, and results in a shift to a higher value of the resonance wavelength (i.e. red shift) [8]. The red shift degrades optical pulses and increases the optical power penalty for the silicon photonic link. It is therefore important to understand the behavior of this thermo-optical effect, and its resulting resonance drift.

In 2012, Li et al. [8] experimentally investigated the red shift effect on the performance of a carrier-injection type silicon ring modulator. They reported a practical upper limit of about 7 dBm on the optical power at the ring input for a 2.5 μm radius ring with quality factor ~ 3000 , and showed that the red shift of this ring follows a parabolic functionality in terms of the input power. To complement these measurements, an analytical model is to be proposed to predict the red shift for a wide range of radii and Q -factors. In 2004, Carmon et al. [9] employed the heat transfer equations to estimate the red shift of the resonance of ultra-high- Q cavities. They demonstrated that the cavity eventually establishes a thermal equilibrium with the surrounding, and that at least one steady state solution exists for the nonlinear heat transfer equation. However, this study was concentrated on ultra-high- Q micro-cavities made of fused silica instead of silicon. More recently, Yan et al. [10] experimentally verified that such nonlinear red shift impacts the power transfer characteristics of silicon channel-dropping ring resonators, and demonstrated that the output power tends to saturate beyond a certain input threshold.

In this work, we adapt the nonlinear dynamic heat transfer differential equation proposed in Reference [9] to silicon photonic waveguide-ring systems. The resulting equation models the temperature rise and its corresponding resonance red shift due to the optical absorption. It shows that the waveguide-ring system eventually reaches a stable thermal and optical equilibrium. Based on this steady state solution, we calculate the temperature change, its associated red shift of the resonance, and finally the input-drop power transfer characteristic of the ring.

Numerical results indicate that the red shift reacts quadratically to the input power, as demonstrated by Li et al. [8] for silicon ring modulators. The input-drop transfer function is not a linear mapping and the slope tends to saturate beyond a certain input power. However, in order to obtain a complete saturation of transfer function, i.e. an optical power limiter, the roll-off tail of the resonance should be steeper than a simple Lorentzian lineshape. This contradicts the analysis done recently by Yan et al [10]. As a way to conciliate their experimental measures and our theoretical results, we approximate the tail of the spectrum of a high- Q ring (outside the 3 dB bandwidth) as a third-order Lorentzian function instead of a simple quadratic one. This approximation agrees with the experimental measurements presented by Lee et al. [11] for the spectrum of a high- Q channel-dropping ring resonator.

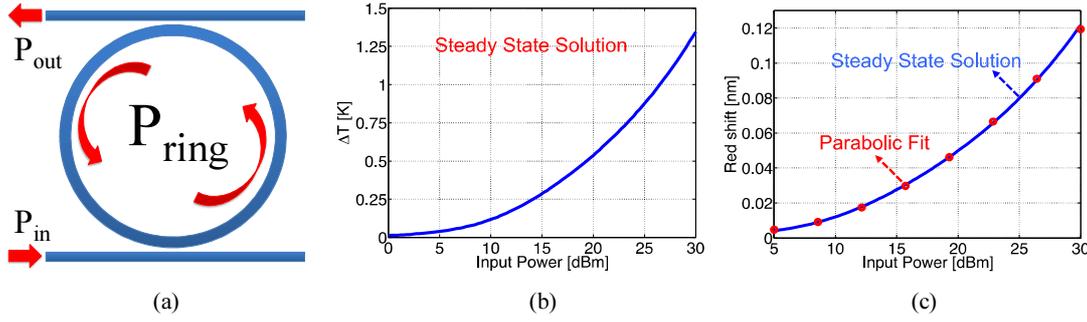


Fig. 1. (a) Schematic of a drop ring resonator. (b) Steady state solution of the temperature change (ΔT) obtained by setting $d\Delta T/dt = 0$ in the dynamic equation. (c) Steady state solution of the red shift (solid) and a parabolic fit (circle).

Our results also indicate that the limiting power factor of a high- Q drop ring resonators starts to be non-negligible at a threshold of about 15 dBm for continuous wave (CW) input light. This is higher than the experimental measurements of Li et al. [8] for ring modulators, but smaller than the nonlinear threshold of 20 dBm generally assumed to calculate optical power budgets [12,13]. Therefore, to conserve this 20 dBm value, thermal stabilization is required to compensate not only temperature changes of the surrounding, but also the ones triggered by the ring itself.

2. Theory and Discussion

We consider a ring resonator coupled to an input and a drop waveguide as shown in Fig. 1(a). The input power is denoted by P_{in} and the output power is denoted by P_{out} . According to the coupled mode theory, as the photons couple from input waveguide into the high- Q ring, the circulation and accumulation of photons right at the resonance leads to the enhancement of power inside the ring [1]. The enhancement factor, defined as the ratio between the optical power inside the ring at the resonance frequency and the one present in the waveguide, is proportional to the finesse F of the resonator. It can be written as $P_{ring}/P_{in} = 2/\pi \times F \times Q/Q_c$ where Q is the loaded quality factor of the ring and Q_c is waveguide-coupling quality factor. Depending on the coupling gap size between the ring and the waveguide (which determines the coupling coefficients), Q/Q_c ranges from 0.5 (critical coupling) to 1, therefore the enhancement factor falls between $1/\pi \times F$ and $2/\pi \times F$. Although it is desirable for the ring to work at the critical coupling, a symmetric design with equal gap sizes between the waveguides and the ring [10] cannot operate at that point. Here, we assume the maximum enhancement ($2/\pi \times F$) in order to account for the worst case. The enhanced power inside the ring triggers nonlinear absorption processes and thus generates heat. Consequently, the spectrum of the resonance tends to drift towards longer wavelengths. This red shift and the increase of the ring's temperature can be related to each other by a linear approximation $x = \lambda_0 \times \alpha \times \Delta T$ [7], where x is the red shift, λ_0 is the original resonance wavelength of the cold ring, α is the thermal coefficient of the resonance ($\alpha = (dn_{si}/dT)/n_{si} \approx 5.338 \times 10^{-5} \text{ K}^{-1}$) [14], and ΔT is the rise in temperature. If we consider a CW input light with a wavelength fixed at λ_0 , then due to the red shift and the Lorentzian lineshape of the resonance, the power inside the ring drops to $P_{ring}/(1+(2/\Delta\lambda \cdot x)^2)$ where $\Delta\lambda$ is the 3-dB bandwidth. On the other hand, the silicon ring has a thermal conductivity $k \approx 6.3 \text{ J.s}^{-1}\text{K}^{-1}$ [10] that helps it relax the temperature increase by conveying some of the heat to the surrounding. Therefore, the rate of temperature change is governed by

$$c_p \rho_s V \frac{d\Delta T}{dt} = \frac{2F P_{in} / \pi}{1 + \left(\frac{2}{\Delta\lambda} \lambda_0 \alpha \Delta T\right)^2} - k \Delta T \quad (1)$$

where $c_p = 0.7 \text{ J.gr}^{-1}\text{K}^{-1}$ is the thermal capacity of silicon, $\rho_{Si} = 2.33 \text{ gr/cm}^3$ is the density of silicon, and V is the volume of the ring. We consider a ring with a radius of about $10 \mu\text{m}$, width about 450 nm , height about 250 nm , λ_0 at $1.564 \mu\text{m}$, and $Q \approx 20000$. These are the numbers that have been reported by Lee et al. [11] for a silicon ring resonator. Thus, the 3dB bandwidth of the ring is about 0.078 nm . With the considered parameters, the enhancement factor of power ($2/\pi F$) turns out to be around 20 dB.

When solving this nonlinear differential equation with all the mentioned parameters, the transient solution shows that the temperature eventually converges to a constant value and a steady state is established. The temperature change and red shift corresponding to this equilibrium are plotted vs. the input power (in dBm) in Figs. 1(b) and 1(c), respectively. As expected, the thermal-drift of the resonance increases with the input power. A parabolic fit is also calculated and plotted in Fig. 1(c) with circles. The steady state solution follows the quadratic behavior very closely [8]. Using the estimated thermal red shift, the roll-off of transmittance $Tr = 1/(1+(2/\Delta\lambda \cdot x)^2)$ is plotted as a solid curve in Fig. 2(a). Note that 1 dB insertion loss from the input port to the drop port is also included in the

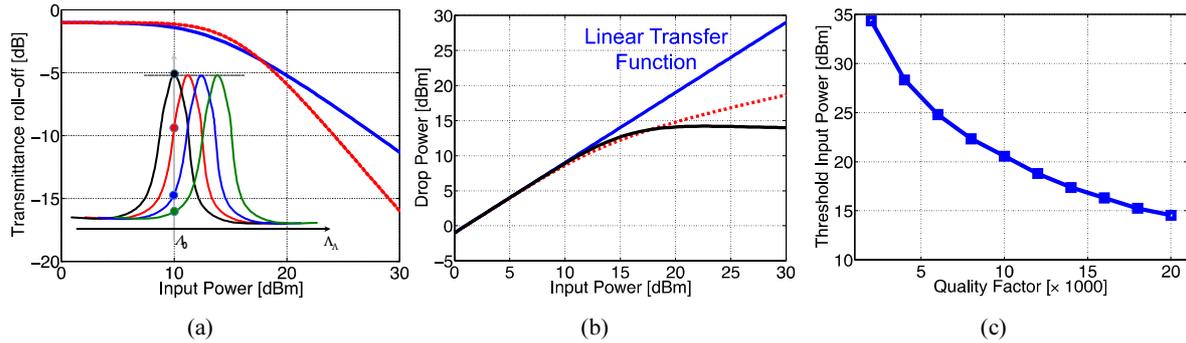


Fig. 2. (a) Calculated roll-off of the transmittance. Inset shows how the red shift and roll-off are related. (b) Input-drop power transfer characteristic of the silicon ring resonator. (c) Estimated input threshold for high- Q and low- Q rings.

calculations to account for possible optical coupling losses. For small input power, the red shift is insignificant compared to $\Delta\lambda/2$ ($= 0.039$ nm) and there is not much reduction in the transmittance. Past a threshold input power of ~ 15 dBm, however, the red shift is significant enough to induce a fast roll-off of transmittance. The input and output powers are related to each other by the relation P_{out} (dBm) = P_{in} (dBm) + Tr (dB). As P_{in} (dBm) increases, the transmittance decreases and this leads to a nonlinear input-output transfer characteristic as shown in Fig. 2(b) by a dotted red curve. The linear transfer function is also plotted in this figure (blue line) as a reference. Despite its nonlinear behavior, the transfer function never saturates at a specific point, i.e. $dP_{\text{out}}/dP_{\text{in}} > 0$ for any given input power. However, a recent measurement by Yan et al. [10] of such high- Q silicon ring resonators indicates that these devices can exhibit a power limiting capability. One possible explanation is that the resonance spectrum of a high- Q ring resonator may have a slightly sharper tail than a Lorentzian lineshape. This was observed in the measurements presented by Lee et al. [11] for a silicon ring with $Q \approx 20,000$. In addition, the slope of the transfer function varies as $dP_{\text{out}}/dP_{\text{in}} = 1 + dTr/dx \times dx/dP_{\text{in}}$, where powers are expressed in dBm and Tr in dB. Noting that if $dTr/dx < 0$ and $dx/dP_{\text{in}} > 0$, the possibility that $dP_{\text{out}}/dP_{\text{in}} = 0$ depends on the magnitude of dTr/dx . Previous measurements [11] show that a third-order Lorentzian function $Tr \approx 1/(1+(2/\Delta\lambda x)^3)$ can model the tail of the resonance more closely. The roll-off of the transmittance, when considering a third-order tail shape, is plotted as a dotted curve in Fig. 2(a). A faster roll-off is seen above the threshold of 15 dBm. This leads indeed to a saturation in the input-drop power characteristic shown in Fig. 2(b) by a solid black curve, in agreement with measurements of Yan et al. [10].

Finally, we consider the behavior of the input threshold with respect to the quality factor of the ring. Since finesse is proportional to the quality factor, low- Q rings are less susceptible to the effects of the thermal drift and thus exhibit a higher nonlinear threshold. This is shown in Fig. 2(c) where the threshold power, defined at 1 dB deviation from the linear transfer function, is plotted vs the quality factor of the ring. This plot demonstrates a nonlinear dependence of the threshold on the Q -factor.

3. Conclusions

By solving a heat transfer differential equation, we analyzed how the self-heating factor of the silicon ring resonators, due to nonlinear absorption, can degrade the device performance through a red shift of the resonance. For high- Q ($>20,000$) channel-dropping rings the optical power threshold, above which significant nonlinearity is observed, is around 15 dBm. Microring devices are capable of behaving as a power limiter if the roll-off tail of their resonance is steeper than a simple Lorentzian lineshape. Two sets of previous measurements indicate that this is the case for high- Q rings.

4. References

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