# Observation of the breakup of a prechirped N-soliton in an optical fiber

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We present what is believed to be the first experimental evidence showing the breakup of a chirped $N$-soliton pulse into an ordered train of fundamental solitons, as predicted by theory. We also present numerical experiments that confirm this phenomenon. Implications for optical communications systems that use chirped pulses are discussed. 

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The propagation of chirped pulses in optical fibers is of great general interest, and the practice of prechirping pulses is becoming increasingly commonplace, particularly in dispersion-managed communication systems.[1] The dynamics displayed by the propagation of chirped pulses is richer and leads to behaviors that differ significantly from the unchirped case. Understanding how prechirping affects pulse propagation in fiber-optic systems is of significant theoretical and practical importance. Recent analytic predictions by Bronski,[2] along with computational evidence by Bronski and Kutz,[3] suggest that a pulse with a strong, symmetric chirp that is injected into a low-dispersion (anomalous) optical fiber will break up into a train of fundamental ($N = 1$) solitons that are ordered by height and propagate away from the initial chirped pulse shape in such a way that the taller ejected fundamental solitons move faster than those of smaller amplitude. In this Letter we report what is believed to be the first experimental evidence of this breakup phenomenon. Previous findings by Friberg and Delong[4] showed that $N$-soliton breakup occurred only under asymmetric perturbations to the pulse. Here we show breakup for a strong, symmetric perturbation. This is an example of symmetry breaking in a nonlinear optical system.

We consider the propagation of a pulse in a lossless low-dispersion anomalous fiber that, in the absence of chirp, propagates as an $N$-soliton ($N > 1$) bound state[5] with some small amount of dispersive radiation. Such an $N$-soliton will undergo a complicated but periodic evolution in which the pulse will appear alternately to break up and recover to its original form. However, the introduction of chirp significantly alters this periodic dynamics by not allowing the pulse to recover its original shape. In particular, the chirp causes an irreversible breakup of the pulse into a train of fundamental solitons that are ejected symmetrically in pairs from the original $N$-soliton at a group velocity determined by the fundamental soliton amplitudes. The maximum number of ejected solitons is determined by the pulse energy and the strength of the initial chirp. For a sufficiently strong chirp, the pulse is completely decomposed into fundamental soliton pairs, whereas for slightly weaker chirps the number of ejected solitons is less than $N$ and some remnant of the original $N$-soliton is left. Specifically, if $M$ fundamental solitons are ejected for a given initial chirp, where $M < N$, then the remnant remaining pulse is an $(N - M)$-soliton pulse.[2] These theoretical predictions, which arise from the Zakharov–Shabat eigenvalue problem associated with the nonlinear Schrödinger equation (NLSE), are surprising since they not only predict pulse breakup but also suggest that the ejected solitons move at a group velocity that is determined by their given amplitudes. This behavior is more similar to Korteveg–DeVries solitons[6] than to typical NLSE solitons whose group velocities are independent of amplitude.[6]

To gain more insight into the evolution dynamics of $N$-soliton breakup we consider the underlying mathematical problem. The evolution of the electric-field envelope in an anomalous dispersion fiber in the presence of the Kerr nonlinearity is given by the NLSE:

$$i\epsilon \frac{\partial Q}{\partial Z} + \frac{\epsilon^2}{2} \frac{\partial^2 Q}{\partial T^2} + |Q|^2 Q = 0\,, \tag{1}$$

where we utilized an $N$-soliton[7] scaling that was appropriate for the low-dispersion regime under consideration when $\epsilon \ll 1$. Thus $Q$ represents the electric-field envelope normalized by the initial peak field power $|E_0|^2$, $T$ represents the physical time normalized by $T_0/1.76$, where $T_0$ is the FWHM of the pulse, and the variable $Z$ is the physical distance multiplied by the parameter $\epsilon$ and divided by the nonlinear length $Z_{\mathrm{NL}} = (\lambda_0 A_{\mathrm{eff}})/(2\pi n_2 |E_0|^2)$. The parameters $n_2 = 2.6 \times 10^{-16}\ \mathrm{cm}^2/\mathrm{W}$, $A_{\mathrm{eff}}$, $\lambda_0$, and $c$ correspond to the nonlinear index coefficient, the effective cross-section area, the free-space wavelength, and the speed of light, respectively. Epsilon is then given by

$$\epsilon^2 = (\lambda_0{}^3 D A_{\mathrm{eff}})/[4\pi^2 c n_2 |E_0|^2 (T_0/1.76)^2]\,, \tag{2}$$

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where $D$ is the chromatic dispersion [in ps/(km nm)] in the optical fiber. We note that the $N$-soliton condition $\epsilon \ll 1$ can be met by propagation in a low-dispersion fiber with high intensities, since $\epsilon \propto D/|E_0|^2$.

As $\epsilon \to 0$, Eq. (1) is in a singular limit, since the dispersive term drops out, and we have at the leading order

$$i\frac{\partial Q}{\partial Z} + \frac{1}{\epsilon}|Q|^2 Q = 0\,, \tag{3}$$

whose solution is simply

$$Q(Z, T) = Q(0, T)\exp[i|Q(0, T)|^2 Z/\epsilon]\,. \tag{4}$$

This leading-order solution suggests and motivates the use of WKB methods, since we have a rapidly varying phase and a slowly varying amplitude. Thus we apply the standard WKB amplitude–phase decomposition $Q(Z, T) = A(Z, T)\exp[-iS(Z, T)/\epsilon]$, which results in the coupled nonlinear equations[8]

$$\frac{\partial A}{\partial Z} = V\frac{\partial A}{\partial T} + \frac{1}{2}A\frac{\partial V}{\partial T}\,, \tag{5a}$$

$$\frac{\partial V}{\partial Z} = V\frac{\partial V}{\partial T} - 2A\frac{\partial V}{\partial T} + \frac{\epsilon^2}{2}\frac{\partial}{\partial T}\left(\frac{1}{A}\frac{\partial^2 A}{\partial T^2}\right), \tag{5b}$$

where we define $V = \partial S/\partial T$. Unfortunately, as we take $\epsilon \to 0$ for the $N$-soliton limit, Eqs. (5) result in an ill-posed problem.[2,3,7,8] Thus the WKB method fails completely owing to the singular behavior of governing equation (1) in the small-epsilon limit. This failure is intimately related to the fact that plane-wave solutions are modulationally unstable in the NLSE.[2,3,7]

Despite this failure to provide an analytic description of the pulse dynamics, we can still characterize the $N$-soliton limit of the NLSE by considering the associated Zakharov–Shabat eigenvalue problem.[2] The resulting analysis predicts the spectral content of the $N$-soliton NLSE. We recall that the discrete spectra, or bound states, of the NLSE correspond to the $N$-soliton solutions.[6] Further, the spectrum of the NLSE is isospectral; i.e., it does not change as a pulse propagates down the fiber. When one is considering the case of the $N$-soliton limit with chirp, the theory predicts that a given number of fundamental solitons will be ejected in pairs from the bulk of the pulse, with taller solitons having a higher group velocity than lower ones. The maximum number of pulses ejected is limited by the number of discrete nonlinear modes that are supported. The spectral description gives some general information about the ejected solitons, but the transformation back to the time domain is extremely complicated. Thus numerical simulations are helpful in elucidating the dynamic evolution of a chirped pulse that is launched into a low-dispersion optical fiber in the anomalous regime.

We begin the simulations at $Z = 0$ with the following strongly chirped pulse profile:

$$Q(0, T) = \text{sech}(T)\exp[2i\,\text{sech}(T)/\epsilon]\,. \tag{6}$$

Computational evidence shows that the same qualitative behavior holds for any initial condition with a single maximum and a large symmetric phase.[3] We note that in the absence of chirp the solution will, in general, consist of an $N$-soliton bound state, where $N$ scales as $1/\epsilon$. A typical realization of the pulse dynamics is presented in Fig. 1, in which we depict the dynamic evolution of the pulse breakup with an initial strong chirp and $\epsilon = 0.1$. Note that the evolution is exactly as predicted from the static Zakharov–Shabat eigenvalue problem: Pairs of fundamental solitons are ejected symmetrically from the middle region, and their pulse speeds are determined by their amplitude. Note that initially ($Z = 4$) a strong oscillatory structure (reminiscent of WKB theory) develops. This is followed by the ordered ejection of the fundamental soliton pulses. By $Z = 10$, the fundamental solitons are well separated and distinct entities that continue to move away from $T = 0$. This behavior is surprising since soliton solutions of NLSE can have arbitrary amplitudes and velocities. Yet the $N$-soliton limit selects solitons with velocities that depend on their amplitude.

This behavior has important consequences for optical fiber applications. In particular, if an intense and chirped pulse is inserted into a low-dispersion, anomalous fiber, it will begin breaking up over a fairly short distance. To demonstrate this principle and verify the theoretical and numerical predictions we perform an experiment in low-dispersion optical fiber. The experiment is performed by propagation of prechirped pulses in a fiber with low anomalous dispersion. The experimental setup is depicted schematically in Fig. 2. A Spectra-Physics Opal optical parametric oscillator provides a train of 160-fs (FWHM) hyperbolic-secant-shaped optical pulses at a repetition rate of 82 MHz and centered at a wavelength of $\lambda_0 = 1550$ nm. These pulses are transform limited and have a spectral bandwidth of approximately 16 nm. The light is coupled into an erbium-doped fiber amplifier (EDFA) that is 20 m long and has a normal-dispersion parameter $D = -25$ ps/(nm km). By starting with low average powers of several milliwatts and then amplifying

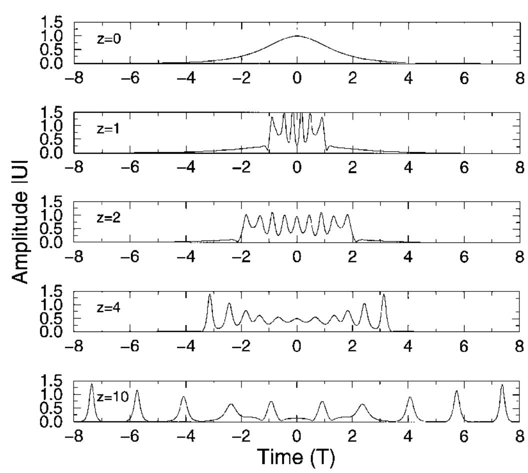


Fig. 1. Numerical simulation of Eq. (1), showing the breakup of a pulse given a strongly chirped initial condition in a low-dispersion, anomalous optical fiber. Note the ejection of pairs of fundamental solitons that propagate away from the center with a speed that is determined by their height.

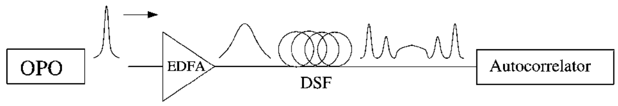


Fig. 2. Schematic of the experimental configuration: OPO, optical parametric oscillator; DSF, dispersion-shifted fiber.

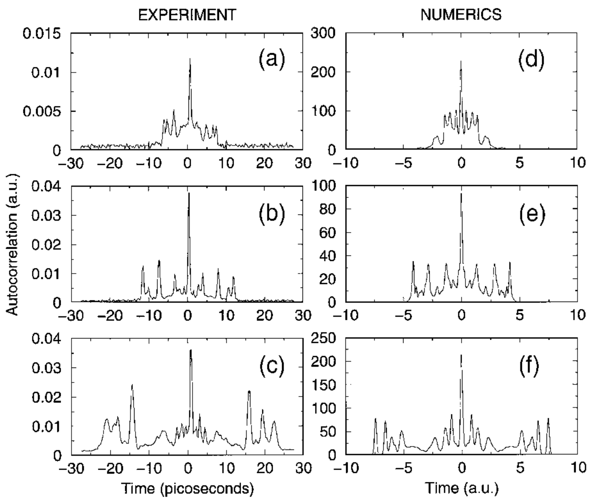


Fig. 3. Left, experimentally achieved autocorrelation as a function of input powers of (a) 9, (b) 13, and (c) 19 mW and, right, autocorrelation generated from the numerical data of Fig. 1 at (d) $Z = 1$, (e) $Z = 2$, and (f) $Z = 4$.

the signal in the EDFA, we are able to keep the spectral width of the pulses essentially unchanged while providing significant chirp. The EDFA saturates near 70 mW, which gives us an adequate range of powers with which to perform the experiment. The optical pulses that emerge from the amplifier have a temporal width of 6 ps (FWHM) and a spectral (3-dB) bandwidth of 20 nm, indicating that they are strongly chirped, with a linear chirp parameter $C$ of the order of 35.[5] This regime of strong chirp and high peak power corresponds to the $\epsilon \ll 1$ limit of Eq. (1). The pulses are then sent to a 220-m section of Corning LEAF dispersion-shifted fiber, which has a dispersion zero at 1520 nm. The core size of this fiber is approximately 10 $\mu$m, so Kerr nonlinearity is weaker than in the standard single-mode fiber and the parameter[5] $\gamma$ is approximately 1.0 $(\text{km W})^{-1}$. At the wavelength of 1550 nm the anomalous dispersion parameter is $D \approx 3$ ps/(nm km). Given these parameters, $\epsilon \approx 0.08$, which is well within the desired regime.

Since it is impractical to keep cutting the fiber length so we can see the same phenomenon as that depicted in Fig. 1, we simply vary the power following the EDFA. With proper scaling, this is equivalent to adjusting the fiber length. Specifically, doubling the peak power is equivalent to decreasing the characteristic breakup length $Z_{\text{NL}}/\epsilon$ by $\sqrt{2}$. So, by adjusting the power, we can effectively reproduce the results of Fig. 1. Finally, since the breakup produces ultrashort pulses, the time-domain behavior is measured by an autocorrelation. In Fig. 3 we depict the experimental results for input powers of 9, 13, and 19 mW alongside the numerical simulation results of Fig. 1, after they are passed through a numerical autocorrelator at $Z = 1$, $Z = 2$, and $Z = 4$, respectively. Note the excellent qualitative agreement between the theoretical and numerical findings and the experiment. This figure demonstrates clearly the phenomenon of pulse breakup owing to a strong, symmetric chirp. We have also included the effects that are due to third-order dispersion and Raman self-frequency shift for the parameter regime considered experimentally and have found that these additional terms have essentially no effect on the breakup process. However, once the breakup has occurred these additional terms act to modify the resulting soliton pulse shapes. We note that the pulse-breakup process in the fiber contributes to timing and amplitude fluctuations that are reflected in the noise of the experimental autocorrelation traces. A more quantitative comparison is difficult, since numerical simulations for $\epsilon = 0.08$ are prohibitively large.

In conclusion, we have demonstrated both numerically and experimentally that a solitonlike pulse with a strong, symmetric chirp that is injected into a low-dispersion, anomalous optical fiber can undergo an irreversible pulse breakup in which a number of fundamental solitons are symmetrically ejected from the pulse, with the tallest solitons moving faster than the smaller ones. This phenomenon can occur over hundreds of meters of fiber and should be avoided in implementations of dispersion-managed transmission systems that rely on chirped solitons.[1]

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